

## DSP : sec 3

### \* Properties of Convolution :-

① Commutation

② Causality  $h(n) = 0, n < 0$

③ stability  $\sum_{-\infty}^{\infty} |h(n)| < \infty$

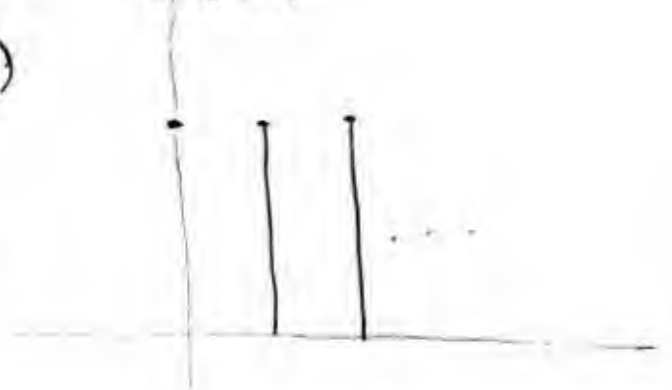
$$* h(n) = U(n) - U(n-a)$$

$$\therefore \sum_{-\infty}^{\infty} h(n) = \sum_{n=0}^{a-1} h(n)$$

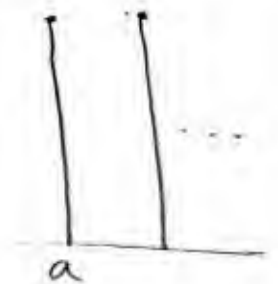
$$= a$$

→ system is stable

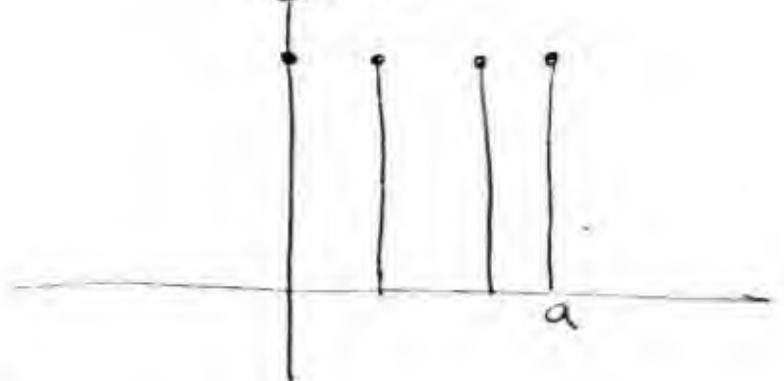
$\Delta U(n)$



$\Delta U(n-a)$



$h(n)$



## Z-Transform

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt \rightarrow \text{Laplace transform (continuous)}$$

$$F(s) = \sum_{n=0}^{\infty} f(nT) e^{-snT} \rightarrow \text{Discrete}$$

$$\text{assume } z = e^{sT}$$

$$F(z) = \sum_{n=0}^{\infty} f(nT) z^{-n}$$

$$F(z) = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$\boxed{\text{ex}} \quad x(n) = n e^{3n} u(n)$$

$$X(z) = \sum_{n=0}^{\infty} n e^{3n} z^{-n}$$

$$= e^3 z^{-1} + 2e^6 z^{-2} + 3e^9 z^{-3} + \dots$$

$$= e^3 z^{-1} [1 + 2e^3 z^{-1} + 3e^6 z^{-2} + \dots]$$

$$= e^3 z^{-1} \left[ 1 + 2 \underbrace{(e^3 z^{-1})}_x + 3 \underbrace{(e^6 z^{-2})}_{x^2} + \dots \right]$$

$$\text{ويكافئ} \rightarrow 1 + 2x + 3x^2 + \dots \quad \boxed{2}$$

$$X(z) = e^3 z^{-1} \frac{1}{(1 - e^3 z^{-1})^2}$$

Note that

$$* 1 + x + x^2 + x^3 + \dots \rightarrow \frac{1}{1-x}$$

$$* 1 + 2x + 3x^2 + \dots \rightarrow \frac{1}{(1-x)^2}$$

$$* 1 - 2x + 3x^2 - \dots \rightarrow \frac{1}{(1+x)^2}$$

another solution

$$X(z) = e^3 z^{-1} + 2e^6 z^{-2} + 3e^9 z^{-3} \dots$$

$$e^3 z^{-1} X(z) = e^6 z^{-2} + 2e^9 z^{-3} \dots$$

$$X(z) - e^3 z^{-1} X(z) = e^3 z^{-1} + e^6 z^{-2} + e^9 z^{-3} \dots$$

$$X(z) [1 - e^3 z^{-1}] = e^3 z^{-1} [1 + e^3 z^{-1} + e^6 z^{-2} + \dots]$$

$$= \frac{e^3 z^{-1}}{1 - e^3 z^{-1}}$$

$$X(z) = \frac{e^3 z^{-1}}{(1 - e^3 z^{-1})^2}$$

$$* x(n) = (-1)^n u(n)$$

$$X(z) = \sum_{n=0}^{\infty} (-1)^n z^{-n}$$

$$= 1 + (-1)z^{-1} + (-1)^2 z^{-2} + (-1)^3 z^{-3} \dots$$

$$= 1 - z^{-1} + z^{-2} - z^{-3} \dots$$

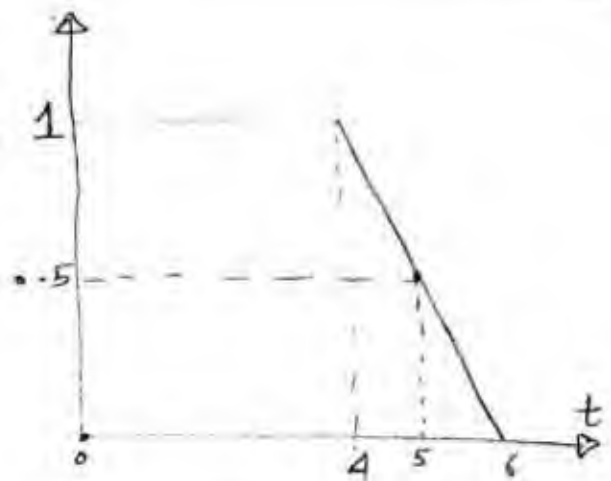
$$= \frac{1}{1 + z^{-1}}$$

\*

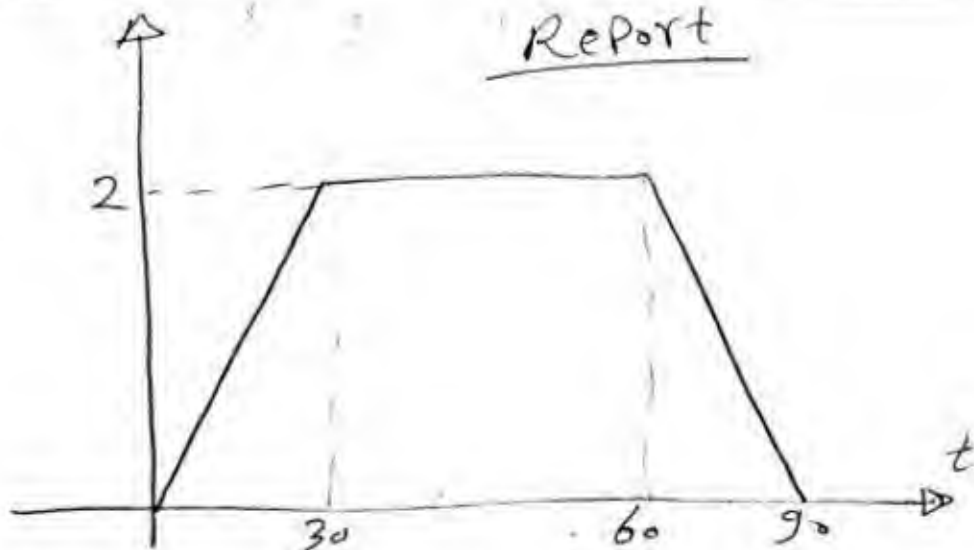
$$x(n) = \{1, 1, 1, 1, 1, 0.5, 0\}$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \dots$$



Report



Properties

$$* Z[a x(n)] = a X(z)$$

$$* Z[X_1(n) \pm X_2(n)] = X_1(z) \pm X_2(z)$$

$$* X_1(n) * X_2(n) \longrightarrow X_1(z) X_2(z)$$

$$* Z[e^{\pm an} x(n)] \longrightarrow X(z) \Big|_{z = z e^{\mp an}}$$

$$* Z[n x(n)] = -z \frac{d}{dz} X(z)$$

$$* Z[a^n x(n)] = X(z) \Big|_{z = \frac{z}{a}}$$

$$* Z(x(n-m)) = z^{-m} X(z)$$

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$$\boxed{\text{ex}} \quad Z((-1)^n u(n))$$

$$\therefore Z[u(n)] = \frac{z}{z-1}$$

$$Z[(-1)^n u(n)] = \frac{z}{z-1} \Big|_{z = \frac{z}{-1}} = \frac{-z}{-z-1}$$

$$= \frac{z}{z+1} = \frac{1}{1+z^{-1}}$$

$$* X(n) = (0.5)^n + (0.25)^n + 3 \delta(n-2)$$

↑  
shift property.

$$X(z) = \frac{z}{z-0.5} + \frac{z}{z-0.25} + 3z^{-2} \quad (1)$$

$$* X(n) = \underbrace{(-1)^n \sin(\omega n)}_{Z[\sin(\omega n)] \Big|_{z = \frac{z}{-1}}} + \underbrace{e^{-2n} \cos \omega n}_{Z[\cos(\omega n)] \Big|_{z = ze^{2n}}}$$

$$* X(s) = \frac{10}{(s+1)(s+2)}$$

$$s \frac{a}{s+1} + \frac{b}{s+2} \Rightarrow x(t) = a e^{-t} + b e^{-2t}$$

$$t \rightarrow nT \text{ (and } T=1) \therefore X(n) = a e^{-n} + b e^{-2n}$$

$$\therefore X(z) =$$

$$* y(n) = x_1(n) * x_2(n)$$

$$x_1(n) = (0.5)^n u(n-1)$$

$$x_2(n) = \delta(n) e^{2n}$$

$$X_1(z) = Z[u(n-1)] \Big|_{z = \frac{z}{0.5}}$$

$$= z^{-1} Z[u(n)] \Big|_{z = \frac{z}{0.5}}$$

$$= \frac{z^{-1} z}{z-1} \Big|_{z = \frac{z}{0.5}} = \frac{1}{2z-1}$$

$$X_2(z) = \dots$$

$$Y(z) = X_1(z) * X_2(z)$$

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